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TRIANGULAR NUMBERS SIMULTANEOUSLY EQUAL TO HEXAGONAL AND STAR NUMBERS

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ABSTRACT

Explicit formulas for the ranks of Triangular numbers, Hexagonal numbers, star numbers satisfying the relation $t_{3,N} = t_{6,h} = S_n$ are obtained.

KEYWORDS: Equality of polygonal numbers, Triangular numbers, Hexagonal numbers, Star numbers.
Mathematics Subject Classification: 11D09, 11D99.

1. INTRODUCTION

The theory of numbers has occupied a remarkable position in the world of mathematics and it is unique among the mathematical sciences in its appeal to natural human curiosity. Nearly every century has witnessed new and fascinating discoveries about the properties of numbers. They form sequences, they form patterns and so on. An enjoyable topic in number theory with little need for prerequisite knowledge is polygonal numbers which is one of the very best and interesting subjects. A polygonal number is a number representing dots that are arranged into a geometric figure. As the size of the figure increases, the number of dots used to construct it grows in a common pattern. Polygonal numbers have been meticulously studied since their very beginnings in ancient Greece. Numerous discoveries arise from these peculiar polygonal numbers and have become a popular field of research for mathematicians. In [1-5], one polygonal number simultaneously equal to an another polygonal number has been studied.

This paper concerns with the study of triangular numbers simultaneously equal to hexagonal and star numbers .

2. NOTATIONS

- Triangular Number of rank N

$$t_{3,N} = \frac{N(N+1)}{2}$$

- Hexagonal Number of rank h

$$t_{6,h} = 2h^2 - h$$

- Star Number of rank n

$$S_n = 6n(n-1)+1$$

3. METHOD OF ANALYSIS

Let N, h, n be the ranks of Triangular, Hexagonal and Star numbers respectively.
The relation

$$t_{3,N} = t_{6,h}$$

leads to

$$N = 2h - 1 \qquad (1)$$

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The assumption $t_{6,h} = S_n$ gives

$$2h^2 - h = 6n^2 - 6n + 1$$

Treating the above equation as a quadratic in n and solving for n, one obtains

$$n = \frac{1}{6}(3 + R) \quad (2)$$

where

$$R^2 = 12h^2 - 6h + 3 \quad (3)$$

On completing the squares on R.H.S in (3), one obtains

$$4R^2 - 3X^2 = 9 \quad (4)$$

where $X = 4h - 1$ (5)

To solve (4), the introduction of the transformations

$$X = P + 4Q\,,\ R = P + 3Q \quad (6)$$

lead to

$$P^2 = 12Q^2 + 9 \quad (7)$$

whose smallest positive integer solution (P_0, Q_0) is

$$P_0 = 21\,,\ Q_0 = 6$$

To obtain the other solutions of (7), consider the pell equation

$$P^2 = 12Q^2 + 1$$

whose general solution is given by

$$\tilde{P}_s = \frac{1}{2} f_s \ ,\ \tilde{Q}_s = \frac{1}{4\sqrt{3}} g_s$$

where

$$f_s = \left(7 + 4\sqrt{3}\right)^{s+1} + \left(7 - 4\sqrt{3}\right)^{s+1} \ ,\ g_s = \left(7 + 4\sqrt{3}\right)^{s+1} - \left(7 - 4\sqrt{3}\right)^{s+1} \ , s = -1, 0, 1, \ldots$$

Applying Brahmagupta Lemma between (P_0, Q_0) and $(\tilde{P}_s\ ,\ \tilde{Q}_s)$, the other integer solutions of (7) are given by

$$P_{s+1} = \frac{21}{2} f_s + 6\sqrt{3} g_s$$

$$Q_{s+1} = 3f_s + \frac{21}{4\sqrt{3}} g_s$$

Substituting the values of P_{s+1} , Q_{s+1} in (6), we have

$$X_{s+1} = \frac{45}{2} f_s + 13\sqrt{3} g_s \ ,\ R_{s+1} = \frac{39}{2} f_s + \frac{45\sqrt{3}}{4} g_s$$

In view of (5), (2) and (1), we get

$$h_{s+1} = \frac{1}{4}\left(\frac{45}{2} f_s + 13\sqrt{3} g_s + 1\right)$$

$$N_{s+1} = \frac{45}{4} f_s + \frac{13}{2}\sqrt{3} g_s - \frac{1}{2}$$

$$n_{s+1} = \frac{1}{6}\left(\frac{39}{2} f_s + \frac{45\sqrt{3}}{4} g_s + 3\right)$$

Note that the values of $t_{3,N_{s+1}} = t_{6,h_{s+1}} = S_{n_{s+1}}$, $s = 0, 2, 4, \ldots$

A few numerical examples satisfying the relations are given in the Table: 1 below

Table: 1 Numerical Examples

s	f_s	g_s	N_{s+1}	h_{s+1}	n_{s+1}	$t_{3,N_{s+1}} = t_{6,h_{s+1}} = S_{n_{s+1}}$
0	14	8	313	157	91	49141
2	2702	1560	60817	30409	17557	1849384153
4	524174	302632	11798281	5899141	3405871	6.95997E+13
6	101687054	58709048	2288805793	1144402897	660721321	2.61932E+18

4. CONCLUSION

To conclude, one may search for the ranks of triples of other special polygonal numbers with the same value.

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